Prof. Thistleton

Recall: We may work with a random variable from the Exponential $Exp(\lambda)$ family of distributions with PDF

$$f(x) = \lambda e^{-\lambda x}$$

Suppose $X \sim Exp(\lambda)$. Define a new random variable $Y = \sqrt(X)$. How is Y distributed?

To get an idea, first run a numerical experiment. Produce random numbers from an exponential distribution with $\lambda = 2$ and then take the square root of each of them. Produce the histogram.

Now to see what the PDF actually is, consider the following. The CDF of an exponential distribution is obtained as $X \sim Exp(\lambda)$, $F_X(x) = 1 - e^{-\lambda x}$ with support on $(0, \infty)$. We then have, for $Y = \sqrt{X}$:

$$F_Y(y) = P(Y \leq y) = P(\sqrt{X} \leq y) = P(X \leq y^2) = F_X(y^2)$$

since the x values range over the positive reals. Thus

$$F_Y(y) = 1 - e^{-\lambda y^2}, \qquad f_Y(y) = \frac{d}{dy}F_Y(y) = 0 - e^{-\lambda y^2}\frac{d}{dy}(-\lambda y^2) = 2\lambda y e^{-\lambda y^2}$$

To confirm this, superimpose this density on a modified relative frequency histogram of your data.

Repeat the process with $X \sim Exp(\lambda = 3)$ and consider the transformation $Y = ln(X)$.