

Working With Continuous Distributions Consider the following examples.

Define

$$f(x, y) = \begin{cases} ax(x + y) & 0 < x < 1, 0 < y < 2 \\ 0 & \text{else} \end{cases}$$

1. Calculate a to make this a legitimate PDF.
2. Calculate the joint CDF $F(x, y) = P(X < x, Y < y)$.
3. Calculate $P(X < .5, 1 < y < 1.5)$.

Define

$$F(x, y) = \begin{cases} (1 - e^{-x})(1 - e^{-y}) & x, y > 0 \\ 0 & \text{else} \end{cases}$$

1. Calculate the joint PDF $f(x, y)$.
2. Calculate the marginal distributions.

Define

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y) & 0 < x, y < 1 \\ 0 & \text{else} \end{cases}$$

1. Calculate the marginal distributions.
2. Calculate the conditional distribution $X|Y = y$.

Define

$$f(x, y) = \begin{cases} 4xy & 0 < x, y < 1 \\ 0 & \text{else} \end{cases}$$

1. Calculate the marginal distributions.
2. Calculate the conditional distributions $X|Y = y$ and $Y|X = x$.

Given a joint distribution $f(x, y)$, we say that X and Y are independent if

$$f(x, y) = f_X(x)f_Y(y)$$

Let X and Y be independent, identically distributed uniform random variables uniformly distributed on $(0, 1)$. (Note: we can write this as $X, Y \stackrel{iid}{\sim} \text{unif}(0, 1)$). Calculate the density of $Z = X + Y$ and verify your result numerically.

Let X and Y be independent, identically distributed random variables exponentially distributed with $\lambda = 1$. (Note: we can write this as $X, Y \stackrel{iid}{\sim} \text{exp}(1)$). Calculate the density of $Z = X + Y$ and verify your result numerically.

Define the convolution of two functions $f(x)$ and $g(x)$ as

$$(f * g)(x) \equiv \int_{-\infty}^{\infty} f(\tau)g(x - \tau)d\tau$$

Use the result that, for independent continuous random variables X and Y with probability density functions $f(x)$ and $g(x)$, we have for the probability mass function of $Z = X + Y$, namely $h(x)$,

$$h(x) = (f * g)(x)$$

to find $h(x)$ if X and Y are *iid* exponentially distributed random variables with common parameter λ .